Goodput Maximization in Opportunistic Spectrum Access Radio Links with Imperfect Spectrum Sensing and FEC-based Packet Protection

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Abstract—We consider a cognitive radio scenario where the communication between two secondary users (SUs) exploits opportunistic spectrum access (OSA) over a wireless channel licensed to primary users (PUs). Assuming a slotted MAC over a single frequency channel and imperfect spectrum sensing, we address the problem of determining the packet size and Forward Error Correction (FEC) coding rate that maximize the SU communication goodput, i.e., the amount of payload bits correctly received in the unit time. Assuming a Markovian model for the PU activity, a saturation regime for the SU, and periodic channel sensing, we find out the mean time between two consecutive packet transmissions and derive approximate analytical expressions, in closed form, which provide upper and lower bounds on the SUs goodput. Such expressions show the dependence of the SU goodput on packet size, FEC coding rate, signal to noise plus interference ratio, amount of resources allocated to sensing, packet overhead, and primary traffic statistics. We provide simulation results showing that the derived analytical expressions are very close to the actual system performance. We also evaluate the sensitivity of the optimal packet size and FEC coding rate pair to the operational conditions represented by the received power and the PU traffic load, showing that the optimal pair is very sensitive to the former, and only moderately affected by the latter.

Index Terms—Cognitive radio, Opportunistic Spectrum Access (OSA), spectrum sensing, packet size, FEC, Markov processes, recurrent events, throughput maximization.

I. INTRODUCTION

The cognitive radio paradigm, [1]–[3], has emerged as a promising approach to increase the efficiency in using the available radio spectrum. In cognitive radio networks (CRNs), wireless terminals learn about the radio environment and exploit this knowledge to adapt their behavior to possibly varying conditions. In the Opportunistic Spectrum Access (OSA) implementation of the cognitive radio paradigm, the “cognition” process is enabled at the physical layer by means of spectrum sensing, [4]–[6]. The so-called secondary users (SUs) acquire knowledge about the status of the channel licensed to primary users (PUs), and opportunistically use the channel if they detect it as currently idle. This channel reuse principle is applicable both in time and space. In the former case, the SU communication happens during time intervals in which the channel is idle; in the latter, it is enabled in spatial regions in which the spectrum is unused [7].

A central issue in any OSA system design is the maximization of the SU performance under a constraint on the performance degradation caused to PUs. In our work, we focus on maximizing the system goodput, defined as the amount of payload bits that a node can correctly receive in the unit time. Specifically, we consider a single-user scenario, where a SU and a PU communication coexist. SU and PU share the same slotted MAC structure. We assume a traffic saturation regime for the SU (i.e., the SU always has data to transmit) and that the PU activity can be modeled by a discrete-time Markov chain (DTMC). Under these assumptions, we investigate how to maximize the goodput experienced by the secondary system, while fulfilling a constraint on the maximum tolerable performance loss of the primary system.

We formulate the SU performance optimization problem taking into account, as a key aspect of the problem, the overhead and redundancy introduced at the different levels of the protocol stack (application, transport, routing, data link control, MAC, and physical layers).

The system parameters we consider for optimizing the SU performance are the packet size and the Forward Error Correction (FEC) coding rate applied to each packet. We show that, to achieve optimal goodput, this pair of parameters should be optimized jointly, on the basis of the conditions in which the SU operates. The operating conditions relevant to our optimization are the Signal to Noise plus Interference Ratio (SNIR) experienced by the SU receiver and the PU traffic statistics.

With respect to previous works, our paper provides the following contributions:

- The computation of the average time between two successive packet transmissions by the SU. We do this by approximating the sequence of the PU traffic as observed by the SU with a Markov chain, and using results from the theory of recurrent events [8].
- The computation of upper and lower bounds on the SU packet error probability, including the effect of using different FEC codes.
- The derivation of analytical expressions for upper and lower bounds on the achievable goodput, which show that the best performance can be obtained combining packet
size (i.e., payload percentage) and FEC coding rate.
- Simulation results are provided to validate our analytical findings and to explore the impact of different relevant parameters on the packet size and FEC code optimization. Summarizing, taking into account the throughput penalty due to spectrum sensing, the sensing performance, and different types of packet overhead, we show that the selection of the SU packet size and the FEC coding rate should be done in a joint manner, in order to maximize the SU goodput.

The paper is organized as follows: Section II discusses works in the literature that are relevant to our problem and clarifies the novelty of our paper. Section III introduces our system model for PU and SU operations, including spectrum sensing and PU traffic modeling assumptions. In Section IV we derive our analytic results in terms of characterization of the average time between two consecutive packet transmissions and average system goodput. In Section V we describe the results of our performance evaluation. Finally, Section VI concludes the paper.

II. RELATED WORK
Designing MAC and scheduling protocols is an important challenge for an effective implementation of CRNs based on DSA. The research in this area has considered a variety of network architecture and system assumptions. An overall overview of the state of the art in this vast field can be found, for instance, in [9]–[12]. Despite this impressive amount of research, it is hard to find works that take into account, at the same time, realistic (i.e. imperfect) spectrum sensing performance, MAC and data link aspects such as the selection of the payload size and the best FEC coding rate, optimizing goodput instead of throughput. We show that, indeed, the above mentioned aspects should be taken into account jointly, to maximize the amount of payload bits transferred through an OSA radio link.

Computing the throughput-optimal sensing time while taking into account realistic sensing performance is the objective of [13]–[15]. In [13], under an unslotted PU channel access and a continuous-time Markovian modeling of the PU activity, optimal channel search times and sensing times are derived which maximise SU throughput. A framework for the optimal scheduling of spectrum sensing and transmission is presented in [14], which also considers adaptive scheduling and cooperative sensing, under an unslotted PU MAC assumption. In [15], Liang et al. assume a slotted PU MAC, with SUs able to synchronize to the PU slot structure. They show that the problem has an optimal sensing time, which yields the highest throughput SUs, under the constraint that primary users are sufficiently protected. A similar analysis is carried out by Wang et al. in [16]. In this paper, we use the same SU MAC and spectrum sensing assumptions of [15], [16].

In [17], the authors consider a time varying traffic-load and channel quality. They show the sub-optimality of performing spectrum sensing at fixed intervals, under such varying conditions. They then propose to adaptively schedule spectrum sensing and data transmissions to channel and traffic conditions, so as to minimize throughput loss and transmission delay. Our contribution differs from [13]–[17] in that these works do not consider the payload/overhead distinction and the presence of FEC, both in terms of overhead bits and the impact on the packet error probability.

Huang et al. investigate in [18], [19], the data capacity, defined as the percentage of time in which SUs can transmit on the channel under stringent constraints on collision probability and overlapping collision time. The assumption on the slotted MAC/PHY organization is similar to our one. Both analytical tools and simulations are used to validate the results, which shed further light into the fundamental trade-off between sensing performance and throughput. This work is relevant to ours as it considers imperfect sensing and focuses on payload bits. However, differently from our work, it does not incorporate the use of FEC to protect SU packets. Indeed, FEC plays an important role and has an interesting interplay with the other system parameters.

The proper dimensioning of payload and overhead in packets is a classical problem in MAC protocol design. Although this problem has been well studied in conventional systems (e.g., WiFi, sensor networks, etc. [20]–[25]), it is hard to find works which address it in new paradigms such as CRNs, DSA and OSA, in conjunction with a realistic modeling of the spectrum sensing activity and other low-level issues which play a key role to obtain an effective, integrated, protocol design. For instance, works such as [26]–[29] address the packet size and payload issues, but do not take accurately into account the effect of imperfect spectrum sensing. In [26], Borgonovo et al. propose a modeling framework to evaluate the performance of SUs in CRNs. Leveraging queuing theory they derive closed form expressions for the throughput and delay of secondary users. The impact of several traffic characteristics (e.g., packet size, primary traffic load, traffic statistics, etc.) is considered on the derived bounds, and they show how the proposed throughput model can be used to optimize the payload length of secondary transmissions. However, their model is based on perfect spectrum sensing (i.e., miss-detection and false alarms are not considered), meaning that secondary users leave/access the channel as soon as the primary presence/absence is detected, without causing any interference or resource waste. In [27], [28], the authors compute the SU goodput. In [28], they compare the impact on goodput of having SU packets with either exponentially distributed payloads or with fixed size. The authors show that, the exponentially distributed payload leads to lower throughput than a fixed-size SU packet stream, even if it has lower blocking and forced termination probabilities. These works assume perfect, and instantaneous, detection of the channel status, as well as the ability to vacate the channel immediately as the PU is back on the channel. In [29], a discrete-time queuing system is used to describe the behavior of a secondary network. The model captures the discontinuous nature of SU transmissions as SUs alternate spectrum sensing and data transmission on the same channel. A slot-by-slot transmission structure and a sense-then-transmit strategy is
adopted, where each slot is fragmented in several periods: a sensing and decision period, a transmission period, and an acknowledgement period. By means of simulations, it is shown that longer packets might increase SUs’ throughput, but lead to a higher number of collisions. In contrast, smaller packets might decrease the probability of collisions, while increasing the overhead of sensing and decision making. Notice that, although [29] considers that sensing inhibits the SU from transmitting in dedicated periods, assuming the alternation of sensing and transmission as in [15], [16] and in our work, it neither accounts for sensing errors, nor for the impact of using different FEC codes.

To the best of our knowledge, our work is the first one to target goodput instead of throughput as the SU performance metric, while combining PHY layer aspects such as spectrum sensing performance with MAC layer issues such as packet size and the choice of the best FEC code.

III. System Model

We consider a scenario where primary, licensed users, coexist with SUs operating in the same wireless frequency band. Primary Transmitters (PTx) use the licensed frequency band to send packets to Primary Receivers (PRx). They operate according to a protocol stack that, in general, is designed without considering the presence of SUs and the possibility to cooperate with them. Secondary Transmitters (STx) can “opportunistically” access the same channel to send their own packets to Secondary Receivers (SRx). The opportunistic channel access protocol should ensure that:

1) When a STx has a packet to transmit, it is able to check if the channel is currently in use by a PTx. In case the channel is busy, the STx refrains from accessing the channel.
2) If the PTx starts using the channel to transmit its own data during a STx packet transmission, the STx will immediately stop transmitting, thus loosing the current packet.

We assume that there cannot be any control information exchange between primary and secondary systems to coordinate transmissions, and that the two requirements described above are fulfilled through the so called “spectrum sensing functionality” of the secondary system. Spectrum sensing enables the STx to detect the primary presence on the channel, inhibiting a packet transmission in case the channel is already in use, or vacating the channel during a SU packet transmission in case new PU transmission begins.

Our objective is the maximization of the goodput experienced by the secondary system, while fulfilling a constraint on the maximum tolerable performance loss of the primary system. The goodput is defined as the amount of payload bits, successfully received per unit time. Particularly, we will show that the selection of the packet size and FEC code pair deeply affects the SU goodput, suggesting that these two parameters should be jointly optimized as a function of received power and PU traffic load. We explicitly account for different types of overhead, which we classify in the following three categories:

a) Redundancy bits for error correction: required to implement the data link layer functionalities. Specifically, we consider the use of FEC codes, which encode bits at a coding rate

\[ K = \frac{b_m}{b_c}, \]  

where \( b_m \) is the number of uncoded bits and \( b_c \) is the length of the codeword. For the sake of simplicity, we assume that the whole packet is encoded at the same rate although, in practical schemes, different portions of a packet may be encoded at different rates.

b) Overhead due to the packet header and trailer: consisting of all the control information that, in any practical communication system, allows the implementation of application, transport, routing, data link control, and MAC protocols. Typical information that may be included in such packet fields are the IP addresses of source and destination, the packet flow identifier, the required QoS indicators, MAC addresses, packet identifiers in PDU MAC fragments, and so on. In this category, we also include a constant (non-depending on the packet size) number of bits, needed for FEC implementation. Additionally, several cyclic redundancy check (CRC) bits may be envisioned to increase the reliability of the data link layer by deciding whether or not to accept a packet. We denote with \( b_h \) the number of bits required in each secondary packet to implement all the above mentioned functionalities and that are independent of the number of payload bits.

c) Overhead due to spectrum sensing: although not consisting of actual transmitted bits, this overhead expresses the percentage of transmission resources that the STx is prevented from using to be able to perform the primary signal detection. This overhead reduces STx transmission efficiency. However, the higher the percentage of transmission resources left for primary user detection, the better its performance are in terms of miss-detection and false alarm probabilities. From the allocation of these resources to the PTx detection or to the STx transmission, it arises the fundamental trade-off described in [15]. We assume that a constant percentage \( \alpha_s \) of transmission resources is not available to SUs due to the implementation of the spectrum sensing functionality.

In the following, we refer to a specific implementation of the physical layer, while fulfilling a constraint on the maximum tolerable performance loss of the primary system. The goodput is defined as the amount of payload bits, successfully received per unit time. Particularly, we will show that the selection of the packet size and FEC code pair deeply affects the SU goodput, suggesting that these two parameters should be jointly optimized as a function of received power and PU traffic load. We explicitly account for different types of overhead, which we classify in the following three categories:

1 Convolutional FEC codes as those we consider in this paper require a constant number of trailing bits, independent of the packet size, to bring the decoder in a regime state.
2 Secondary systems in which it is possible to operate the spectrum sensing functionality on the same set of resources used for transmission are covered by our framework. In this case, the spectrum sensing functionality has no impact in terms of overhead.
to scenarios with different PHY characteristics. Similarly to [15], we consider a system in which PUs and SUs share the same slotted MAC structure, leaving the study of the case with absence of synchronization between SU and PU, or with an unslotted PU MAC, to our future work. PUs transmit packets of variable length, starting at the beginning of a slot, and corresponding to an integer number of slots (See Subsection III-A for our assumption on the PU traffic statistics). We assume that SUs are equipped with a half-duplex radio transceiver, which cannot transmit and receive (and therefore, sense) at the same time on the same frequency band. A sense-then-transmit strategy is hence employed by the SU in each slot. The percentage of resources made available for transmission due to the implementation of spectrum sensing consists of small sensing intervals placed at the beginning of each slot\(^3\) (see Fig. 1). SUs transmit packets with size corresponding to a fixed integer number of slots. In our set-up, the interruption of an SU packet transmission due to the detection of the PU presence determines the loss of the whole SU packet. On one hand, the risk of losing the whole packet due to a PU return can be reduced through a decrease of the packet size. On the other hand, increasing the packet size yields a reduction of the overhead percentage. Finding the best compromise between these two effects allows to achieve goodput-optimal performance. In this paper, we consider the single user case, for both primary and secondary systems, leaving the analysis of the multi-user case to a future work.

As showed in Fig. 1, if the channel is detected as idle during the sensing interval, the rest of the slot is used for transmission; otherwise, if a primary signal is detected, the transmission is interrupted, causing a loss of the current packet. We point out that SU transmissions are periodically suspended (in the initial portion of each slot) for a pre-determined amount of time in order to perform spectrum sensing; we are implicitly assuming that the receiver is able to recover the signal tracking at the end of the sensing interval at the physical layer. The assumption is justified in a scenario in which the sensing interval lasts for a very short time, sufficiently short to let the receiver keep the synchronization, even in phase, with the signal. On the other hand, interruptions occurring upon detection of the PU signal do cause the loss of the whole packet, because the time interval after which the SU will have access again to the channel is unknown to the SU receiver and much longer (several time-slots) than the sensing intervals. We remark that the slot duration is not, for us, a parameter to optimize, since

\[ b' = (1 - \alpha_s) \tau_s B = (1 - \alpha_s) b. \]  

With an overhead of \(b_h\) bits, we obtain the following expression for \(b_m\):

\[
b_m = K \cdot (Mb' - b_h) = K \cdot (Mb (1 - \alpha_s) - b_h).\]

In the following, we describe our assumptions for modeling the primary activity on the channel and the spectrum sensing performance in the SUs.

### A. Primary system model

The primary user activity is represented by an alternating pattern of periods during which either the primary uses the channel to transmit (busy periods) or the channel is unused (idle periods). We denote with \(x[k]\) the state of the channel during slot \(k\). Specifically, \(x[k] = 1\) means that the channel is in use by the PU, and \(x[k] = 0\) that the primary is not transmitting. We model \(x[k]\) as a two state DTMC, with steady-state probabilities defined as \(p_0 \triangleq P(x[k] = 0)\) and \(p_1 \triangleq P(x[k] = 1)\), and transition probabilities

\[
\begin{align*}
p_{00} & \triangleq P(x[k] = 0/x[k-1] = 0) \\
p_{01} & \triangleq P(x[k] = 1/x[k-1] = 0) \\
p_{10} & \triangleq P(x[k] = 0/x[k-1] = 1) \\
p_{11} & \triangleq P(x[k] = 1/x[k-1] = 1).
\end{align*}
\]

According to this assumption, the busy and idle periods follow a geometric distribution, with average sojourn times \(\mu_0\) and \(\mu_1\) (measured in number of steps) in the idle or busy state respectively given by (see e.g. [30]) \(\mu_0 = 1/(1-p_{00})\) and \(\mu_1 = 1/(1-p_{11})\).

### B. Spectrum sensing model

The periodic sensing activity of the primary presence on the channel enables the STx to avoid interfering with primary transmissions. In our model, we consider the case of imperfect sensing, which means that the primary activity detection may fail. Specifically, we may incur a miss-detection, when the detector does not reveal the primary transmission on the channel, or a false alarm, when the detector reveals a primary signal on the channel, although the PTx is not transmitting.
Imperfect sensing of the channel affects both primary and secondary system performance. In this work, we assume that the constraint on the impairments caused to the primary system is in the form of a prescribed miss-detection probability. This key constraint could be imposed in the form of alternative metrics like PU packet error rate (PER), PU bit error rate (BER), amount of interference (AOF), and so on. The reason to use the miss-detection probability is that all these performance metrics depend on it, but it also has the advantage of an immediate and simple functional relation with the SU performance. In the following, we indicate with \( y[k] \) the decision of the primary signal detector in the \( k \)-th slot. Each element of the sequence \( y[k] \) can be either “0”, if the STx has decided that the channel is idle, or “1”, if it appears to be busy. The performance of the detector are expressed in terms of false alarm and miss-detection probability, defined as:

\[
\begin{align*}
    p_{fa} &= P(y[k] = 1/x[k] = 0) \\
    p_{md} &= P(y[k] = 0/x[k] = 1).
\end{align*}
\] (6)

We are not interested in writing explicit equations for the detector since these equations are implementation specific. The important aspects, at least for the class of hard detectors which is of interest for us, are that: i) A detector compares a suitable test statistic, which is a function of a set of signal readings, with a threshold; ii): For a given technology, setting the threshold determines the false alarm and miss-detection probabilities: increasing the threshold lowers \( p_{fa} \) but increases \( p_{md} \); iii): Since increasing the sensing interval allows to take more signal samples, the sensing performance \( p_{fa} \) and \( p_{md} \) are also function of the length of the time interval \( \alpha_sT_s \), besides the primary signal strength.

### IV. Secondary User Goodput Maximization

#### A. The “perceived” primary traffic

For the evaluation of the SU goodput, it is important to properly take into account that the STx decides to access the channel on the basis of its perception of the channel idle/busy states. Since we assume that the SU has always information to transmit, it performs spectrum sensing at any time slot. We indicate with \( y[k] \) the sequence of decisions made by the detector. The statistical properties of the sequence of sensing outcomes \( y[k] \) are determined by the transition probabilities of the DTMC \( x[k] \) and the false alarm and miss-detection probabilities of the detector. For the sequence \( y[k] \), due to the sensing performed at each step, it can be showed that, in general, the Markov property does not hold. For small values of \( p_{md} \) and \( p_{fa} \), however, the dependence of the state in the current step from the past history is negligible. In such a case, the modified DTC retains, approximately, a Markovian behavior. In practical terms, this means that the distribution of the length of runs of consecutive “0” or “1” states is very well approximated by a geometric distribution. In the following, we will treat \( y[k] \) as a DTMC. To indicate its steady-states and transition probabilities, we will use the following notation:

\[
\begin{align*}
    q_0 &= P(y[k] = 0) & q_{00} &= P(y[k] = 0/y[k-1] = 0) \\
    q_1 &= P(y[k] = 1) & q_{10} &= P(y[k] = 0/y[k-1] = 1) \\
    q_{01} &= P(y[k] = 1/y[k-1] = 0) & q_{11} &= P(y[k] = 1/y[k-1] = 1). \quad (7)
\end{align*}
\]

Using standard Bayesian analysis arguments, we can derive the following expressions of the steady states and transition probabilities of the chain \( y[k] \), which are function of the corresponding probabilities for the actual channel use sequence \( x[k] \), and the miss-detection and false alarm probabilities of the detector:

\[
\begin{align*}
    q_0 &= p_0 (1-p_{fa}) + p_1 p_{md} \\
    q_1 &= p_0 p_{fa} + p_1 (1-p_{md}) \quad (8)
\end{align*}
\]

and

\[
\begin{align*}
    q_{00} &= p_0 (1-p_{fa}) (p_{11} p_{md} + p_{00} (1-p_{fa})) \\
          &+ p_1 p_{md} (p_{11} p_{md} + p_{01} (1-p_{fa})) \\
    q_{01} &= p_0 (1-p_{fa}) (p_{00} p_{fa} + p_{01} (1-p_{md})) \\
          &+ p_1 p_{md} (p_{10} p_{fa} + p_{11} (1-p_{md})) \\
    q_{10} &= p_0 p_{fa} (p_{00} (1-p_{fa}) + p_{01} p_{md}) \\
          &+ p_1 (1-p_{md}) (p_{10} (1-p_{fa}) + p_{11} p_{md}) \\
          &+ p_0 p_{fa} + p_1 (1-p_{md}) \\
    q_{11} &= p_0 p_{fa} (p_{00} p_{fa} + p_{01} (1-p_{md})) \\
          &+ p_1 (1-p_{md}) (p_{10} p_{fa} + p_{11} (1-p_{md})) \\
          &+ p_0 p_{fa} + p_1 (1-p_{md}). \quad (9c)
\end{align*}
\]

#### B. Packet transmission rate

In the following, we restrict our attention to the case of a SU working in a saturation regime. From the description of our system model, it is clear that the transmission of a SU packet can be completed if and only if the STx detects a sequence of \( M \) consecutive idle slots on the channel.

We define the packet transmission rate \( R_{Tx} \), as the number of packets transmitted in a given time interval by the SU, divided by the duration of that interval. \( R_{Tx} \) represents “how often” the STx manages to complete the transmission of a packet, given that each PU signal detection stops its transmissions. To find \( R_{Tx} \), we resort to the theory of recurrent events [8]. A known result of this theory characterizes the behavior of the waiting time (i.e. the number of trials or, in our case, time slots) required to observe a run of a given length of consecutive steps in the same state, of a two-state DTMC with known transition probabilities. Such number of trials is a
random variable that has been studied in [32]. To determine the 
average time interval between the transmission of consecutive 
packets, let us consider the last slot of a packet transmission 
and denote it, without loss of generality, with the step index 
0. The fact that the slot has been used for transmission from 
the SU yields that we should consider the state of the chain 
in step zero as idle, i.e. \( y[0] = 0 \). Now, the time one has to 
wait to transmit another packet is exactly the waiting time to 
observe (for the first time) a new run of \( M \) consecutive zero 
states. We remark that it is important to specify from which 
state “we are starting to wait for the new run”. In fact, the 
statistics of the waiting time when starting to wait from a 
“0” are different from those of the waiting time when starting 
from a “1”. Clearly, we are interested in the former of the two. 
In particular, from [32], eq. (2.17), we can express the mean 
waiting time, measured in number of time slots, as 
\[
\bar{t}_M = \frac{1 - (1 - q_{01})^M}{q_{01} q_{10} (1 - q_{01})^{M - 1}} \left( q_{01} + q_{10} \right). 
\]  
(10)
This expression gives the average time the SU has to wait for 
managing to transmit a packet with the duration of \( M \) slots. 
The inverse of this average waiting time tells us how many 
packets the SU manages to transmit per unit time, i.e. the SU 
packet transmission rate \( R_{Tx} \). In packets per time slot, we have 
\[
R_{Tx} = \frac{1}{\bar{t}_M} = \frac{q_{01} q_{10} (1 - q_{01})^{M - 1}}{1 - (1 - q_{01})^M} \frac{1}{(q_{01} + q_{10})}. 
\]  
(11)
Notice that (11) depends both on the primary traffic statistics 
and on the detector performance, since \( q_{01} \) and \( q_{10} \) are 
parameters of the DTMC representing the perceived traffic, which 
depend on the primary traffic and the detector by (9b) and 
(9c). In the following subsection we model the performance 
at the receiver, which allow us to compute the average packet 
correct reception rate.

C. Packet reception rate

The SU communication goodput is determined by the percentage 
of the transmitted packets that the receiver manages to 
correctly decode. The packet error rate, in general, depends on 
the particular FEC code with which packets are protected and 
on the SNIR experienced by the receiver during the reception 
of the packet. In the following, we restrict our attention to 
convolutional codes\(^6\). The reason is their high flexibility in 
terms of the encoded message length, which allows us to 
“scan” the packet size domain with an arbitrary granularity 
and range, while leaving unchanged the ratio between message 
and codeword length (except for the trailing bits, which do not 
depend on the message length). More details on the chosen 
codes will be given in Section V. For a given convolutional 
encoder of rate \( K \) (whose structure does not depend on 
the encoded message length), assuming that the SNR at the 
SRx remains constant during the whole packet reception, the 
packet error probability, which we denote as \( P_e \), is uniquely 
determined by the SNR value and by the packet length \( M \). 
\[
P_e = \bar{P}_e(K, M, \sigma^2), 
\]  
(12)
where \( P_{rx} \) is the power received by the SRx and \( \sigma^2 \) is the noise 
power. In our scenario, we must take into account that there is 
a non-zero probability that in one, or more, of the \( M \) time-slots 
used by the SU to transmit the packet, the PU does, in fact, use 
the channel, while the SU doesn’t detect its transmission due 
to sensing errors (miss-detections). Therefore, the SU packet 
error probability depends on the pattern of such actual channel 
uses. Depending on the specific code, (12) may or may not be 
available in closed form, but can always be found by numerical 
simulations and tabulated for each value of interest of the 
SNR. In other words, it can safely be assumed as a known 
characteristic of the code that only depends on the values of 
\( K \) and \( M \).

We now indicate with \( S = \{ s_0, \ldots, s_{2^M-1} \} \), the set of 
\( 2^M \) possible sequences of states of the PU during the 
\( M \) time slots, where \( s_i \) is (without loss of generality) the binary 
representation of the number \( i \), \( \forall i \in \{0, \ldots, 2^M - 1 \} \). We 
observed that, while receiving a packet transmitted by the 
STx, the SRx could experience an interfering signal coming 
from the PTx which may follow any of the distinct patterns 
represented by the sequences introduced above, although with 
different probabilities. The PTx-vs-PRx interference, \( P_{int} \), 
generated by the (undetected) primary transmission in a subset 
of the SU packet time-slots, is summed to the noise term, 
thus increasing (in those time slots) the noise plus interference 
power term. It is clear that each of the possible interference 
patterns corresponds to a distinct SU packet error probability. 
We denote the packet error probabilities for the patterns in 
\( S \) with \( P_e(s_i), \forall i \in \{0, \ldots, 2^M - 1 \} \), where the dependency 
on \( K, M, \sigma^2 \), and \( P_{int} \) has been dropped, since they are 
intended to be fixed parameters in the following derivations.

We now define \( \hat{P}(s_i) \) as the posterior conditional probabil-
ity of the sequence of states \( s_i \), i.e. the probability that, given 
that the SU has succeeded in transmitting over a sequence 
of \( M \) slots (i.e., it has transmitted a packet), the interference 
from the PU, in those \( M \) slots, has followed the pattern \( s_i \). 
The average packet error probability, for a given packet size 
\( M \) and error correction coding rate \( K \), is given by 
\[
P_e(K, M) = \sum_{i=0}^{2^M-1} P_e(s_i) \cdot \hat{P}(s_i). 
\]  
(13)
We remark that the \( \hat{P}(s_i) \) are conditional probabilities, with 
the conditioning event being a sequence of \( M \) idle state 
detections in the chain \( x[k] \).

The problem with (13) is that, except for the case of \( M \) 
consecutive channel idle states \( s_0 = [0, \ldots, 0] \), corresponding 
to \( M \) consecutive correct decisions, expressions for the terms 
\( P_e(s_i) \) are not available. \( P_e(s_0) \) coincides with (12), i.e. the 
performance of the code in the presence of a constant SNR, 
and is hence available, whereas the remaining \( 2^M - 1 \) terms 
are too many (even with moderately large values of \( M \)) to 
be computed for all the desired SNR values. To proceed with 
our analysis, we resort to lower and upper bounds for the 
packet error probability. We can readily say that the packet 
error probability (13) is lower bounded by \( P_e(s_0) \), which 
corresponds to the performance in the absence of PU signal 
in all the \( M \) time-slots, since it represents the characteristic of 

\(^6\)All our analytical results can be applied also to block codes.
the chosen code, i.e. (12). On the other hand, an upper bound on the PER can be obtained bounding the probabilities $P_e(s_i)$, $i = \{1, \ldots, 2^M - 1\}$, with the value $^7$ 1, i.e. $P_e(s_i) < 1$. We have hence
$$P_e(s_0) \leq P_e(K, M) = P_e(s_0) \tilde{p}(s_0) + \sum_{i=1}^{2^M-1} P_e(s_i) \tilde{p}(s_i)$$
$$< P_e(s_0) \tilde{p}(s_0) + 1 \cdot \sum_{i=1}^{2^M-1} \tilde{p}(s_i)$$
$$= P_e(s_0) \tilde{p}(s_0) + (1 - \tilde{p}(s_0)). \quad (14)$$

To derive a suitable expression for $\tilde{p}(s_0)$, let us denote with $x \triangleq [x_1, \ldots, x_M]$ and $y \triangleq [y_1, \ldots, y_M]$ the vectors composed by the sequences of states of the PU channel use and perceived channel use, respectively, during the transmission of a SU packet of length $M$ time slots. Notice that, without loss of generality, we have use the time indexes $1, \ldots, M$ to identify such time slots. The posterior conditional probability of the sequence $s_0$ is given by
$$\tilde{p}(s_0) = \frac{p(x = s_0, y = s_0)}{p(y = s_0)}. \quad (15)$$

We now indicate the generic sequence $s_i \in \{0, 1\}^M$ defined above with $s_i \triangleq \{s_{i1}, \ldots, s_{iM}\}$, and define the function $\phi$ of the generic state $s \in \{0, 1\}$ as
$$\phi(s) = \begin{cases} 
(1 - p_{fa}) & \text{if } s = 0 \\
 p_{md} & \text{if } s = 1.
\end{cases}$$

It can be showed that the probability $p(y = s_0)$ is given by
$$p(y = s_0) = p_0 (1 - p_{fa}) \sum_{i=0}^{2^M-1} \prod_{k=2}^{M} p_{s_{ik}} s_{ik} \phi(s_k)$$
$$+ p_1 p_{md} \sum_{i=2^M-1}^{M} \prod_{k=2}^{M} p_{s_{ik}} s_{ik} \phi(s_k). \quad (16)$$

where we have used the index the number $i$ as an index into the set $\{0, 1\}^M$ according to the definition of $s_i$ given above, and used the couples of states $s_{i1}, s_{i2}$ as indices into the set of the transition probabilities $\{p_{00}, p_{01}, p_{10}, p_{11}\}$.

The joint probability $p(x = s_0, y = s_0)$ is the probability that a sequence of $M$ idle states occurs in the PU channel uses, and each of them is correctly identified by the SU. Hence
$$p(x = s_0, y = s_0) = p_0 p_{00}^{M-1} (1 - p_{fa}) M. \quad (17)$$

Combining (17) and (16) into (15), one can compute a suitable expression for the upper bound in (14).

We now define the packet reception rate $R_{Rx}$, as the average number of packets received (i.e. correctly decoded) by the SRx per unit time. Combining the packet error probability with (11), we obtain the following expression of $R_{Rx}$ in packets per time-slot:
$$R_{Rx} = R_{Tx} (1 - P_e(K, M)) = \frac{q_{01} q_{10} (1 - q_{01})^{M-1} (1 - P_e(K, M))}{1 - (1 - q_{01})^M} \cdot \frac{1}{q_{01} + q_{10}}. \quad (18)$$

$^7$A tighter bound could be obtained using, in place of 1, the PER value corresponding to (12), with signal to noise ratio $P_{rx}/(\sigma^2 + P_{in}^2)$. The simpler bound obtained with 1 fits, however, to our purposes.
We have analysed the goodput performance of an OSA-based cognitive radio system over a single frequency channel. The SU access is controlled by a periodic sense-then-transmit protocol over a single frequency channel. This protocol is characterized by a sensing time and a transmission time, which are determined by the traffic load and the signal-to-noise ratio (SNR) of the SU channel. The SU channel is assumed to be characterized by a constant signal power, which ranges from -10 dB to 10 dB, and the (normalized) interference power, which ranges from -13.2 to 6.9 dB.

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strategy, with periods coinciding with the slots of a PU slotted MAC, allowing for channel release in case a PU starts using the channel during a SU packet transmission. Our analysis considered the presence of sensing errors and overhead, including FEC redundancy. Under a Markovian PU activity model and a SU traffic saturation regime, we have obtained expressions of upper and lower bounds on the packet error probability in the presence of miss-detections, and, finally, of upper and lower bounds on the average goodput. Our results suggest that FEC codes and packet size should be chosen taking into account the received power, interference power, and PU traffic statistics.

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REFERENCES